Theoretical Analysis for Cross-edge Computation Offloading

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1 Theoretical Analysis

1.1 Feasibility with Deterministic Queue Bounds

As mentioned before, mobile devices with a *finite* battery capacity can implement CCO algorithm to obtain a near optimal solution of $\mathcal{P}_1$. This subsection first gives the upper bound of battery capacity of mobile devices, then verifies the feasibility of CCO algorithm by proofing that the constraint (6) can always be satisfied.

Lemma 1 *(Finite Battery Energy Level Implementation)* $\forall i \in \mathcal{N}$*, if* $\psi_i(0) \leq \theta_i + E_{i,h}^{max}$*, then CCO algorithm yields* $\psi_i(t) \leq \theta_i + E_{i,h}^{max}$ *for every* $i \in \mathcal{N}, t \in \mathcal{T}$*.*

Proof 1.1 $\forall t \in \mathcal{T}$*, for the ith mobile device, (i) suppose* $\theta_i < \psi_i(t) \leq \theta_i + E_{i,h}^{max}$*, according to (10), we have* $\psi_i(t+1) \leq \psi_i(t) \leq \theta_i + E_{i,h}^{max}$ *because* $\alpha_i^\star(t) = 0$*. Otherwise, (ii) if* $\theta_i \geq \psi_i(t)$*, we have* $\psi_i(t+1) \leq \psi_i(t) + \alpha_i^\star(t) \leq \theta_i + \alpha_i^\star(t) \leq \theta_i + E_{i,h}^{max}$*. q.e.d.*

The following proposition demonstrates that (6) can always be satisfied under CCO algorithm.

Proposition 1 *(Feasibility with Finite Queue Bounds) By solving* $\mathcal{P}_2$ *with CCO algorithm, the constraint (6) is not violated, which means the solution for* $\mathcal{P}_1$ *obtained by CCO algorithm is feasible.*

Proof 1.2 *For the ith mobile device, (i) suppose* $\psi_i(t) \geq E_{i,all}^{max}$*, then we have* $\max_{\mathbf{I}_i(t)} \big\{\epsilon_i^l + \sum_{j\in\mathcal{M}} \epsilon_{i,j}^{tx}(t) \cdot I_{i,j}(t)\big\} \leq E_{i,all}^{max} \leq \psi_i(t)$*. Otherwise, (ii) if* $\psi_i(t) < E_{i,all}^{max}$*, the optimal edge site-selection problem has the optimal decision* $\mathbf{I}_i^\star(t) = \mathbf{0}$*, i.e.,* $D_i(t) = 1$*, which can be obtained by the lower bound of* θ_i*. q.e.d.*

The above proof uses the fact that if the battery energy level is not sufficient for sending the offloading request (which means the local execution part cannot be finished successfully), i.e., $\psi_i(t) < \epsilon_i^l$, the total energy consumption is zero.

1.2 Asymptotic Optimality of CCO algorithm

In order to analyze the optimality of CCO algorithm, inspired by [1] and [2], we first define problem $\mathcal{P}_3$ as

$$\begin{aligned}
\mathcal{P}_3: \quad \min_{\forall i, \mathbf{I}_i(t), \alpha_i(t)} \quad & \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\Big[\sum_{i\in\mathcal{N}} \mathcal{C}(\mathbf{I}_i(t))\Big] \\
s.t. \quad & (2),\ (9),\ (11) \\
\lim\nolimits_{T\to\infty} \quad & \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\big[\zeta_i(t) - \alpha_i(t)\big] = 0,
\end{aligned}$$

where $\zeta_i(t) \triangleq \epsilon_i^l + \sum_{j\in\mathcal{M}_i(t)} \epsilon_{i,j}^{tx}(t) \cdot I_{i,j}(t)$ is the energy consumption of the ith mobile device. Actually, $\mathcal{P}_3$ is a relaxed version of $\mathcal{P}_1$, i.e., $G_{\mathcal{P}_3}^\star \leq G_{\mathcal{P}_1}^\star$, where $G_{\mathcal{P}_3}^\star$ and $G_{\mathcal{P}_1}^\star$ are the optimal objective functions of $\mathcal{P}_3$ and $\mathcal{P}_1$, respectively.

Lemma 2 *(Existence of **optimal** $(\mathbf{A}(t), \mathbf{E}^h(t))$**-only policy)** For an arbitrary $\delta > 0$, there exists a stationary and randomized policy π^* for $\mathcal{P}_3$, which satisfies*

$$\mathbb{E}\big[\sum_{i=1}^{N} \mathcal{C}^*(\mathbf{I}_i(t))\big] \leq G^{\star}_{\mathcal{P}_3} + \delta,$$
$$\big|\mathbb{E}[\zeta_i^*(t) - \alpha_i^*(t)]\big| \leq \varpi \cdot \delta,$$

where ϖ is a scaling constant.

Proof 1.3 *The proof can be obtained by Theorem 4.5 in [2]. Similar proof can be found in [1].*

Now we dicuss the Performance of CCO algorithm.

Theorem 1 *(Performance of CCO algorithm) Under the CCO algorithm implemented with any parameter $V > 0$, we have:*
(i)

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}_{CCO}\big[\sum_{i=1}^{N} \mathcal{C}(\mathbf{I}_i(t))|\mathbf{\Theta}(t)\big] \leq G^{\star}_{\mathcal{P}_1} + \frac{C}{V}, \tag{1}$$

where $\mathbb{E}_{CCO}[\cdot]$ is the expectation obtained by CCO algorithm. It means that the optimization goal deviates by at most $O(\frac{1}{V})$ from the optimum.
(ii) if $\forall i \in \mathcal{N}, E^{max}_{i,all} \leq \psi_i(0) \leq \theta_i + E^{max}_{i,h}$, then

$$E^{max}_{i,all} \leq \psi_i(t) \leq \psi_i^{max} \triangleq \theta_i + E^{max}_{i,h}, t \in \mathcal{T}, i \in \mathcal{N},$$

which means the maximum queue buffer size (battery energy level) ψ_i^{max} is $O(V)$.

Proof 1.4 *(i) By utilizing Lemma 1 and Lemma 2, it is easy to obtain that*

$$\Delta(\mathbf{\Theta}(t)) + V \cdot \sum_{i=1}^{N} \mathcal{C}(\mathbf{I}_i(t)) \leq V \cdot G^{\star}_{\mathcal{P}_3} + C,$$

where the left is solved by CCO algorithm. Therefore, by taking the time average expectation of (1.4), we have $G^{CCO}_{\mathcal{P}_2} \leq G^{\star}_{\mathcal{P}_3} + \frac{C}{V}$, where $G^{CCO}_{\mathcal{P}_2}$ is the optimal solution obtaiend by CCO algorithm By utilizing that $G^{\star}_{\mathcal{P}_3} \leq G^{\star}_{\mathcal{P}_1}$, (1) can be obtained. (ii) The proof can be obtained with the assistance of Lemma 1. Similar proof can be consulted in Theorem 4.5 of [3].

Apperantly, $\forall i \in \mathcal{N}, \theta_i$ increrases along with the growth of V, which means a larger batery capacity is needed, and a longer time horizon is necessary for the convergence of CCO algorithm and system stability. However, higher asymptotic optimality can be achieved.

References

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